电磁学期末复习

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$$\nabla \cdot \boldsymbol{D} = \rho_f \qquad \oiint \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = \iiint \rho_f \mathrm{d}V$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint_{S_C} \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \oiint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\nabla \times \boldsymbol{H} = \vec{i}_f + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \iint_{S_C} \left(\vec{i}_f + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot \mathrm{d}\boldsymbol{S}$$

目录

第一部分　知识点集合

1　真空中的静电场与静磁场

1.1　静电场

- 电场力

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

- 电场

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$\vec{E} = -\nabla U$$

- 电势

$$U = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}_i|}$$

$$U_{ab} = \int_a^b \vec{E}\cdot \mathrm{d}\vec{l}$$

$$U_P = \int_P^\infty \vec{E}\cdot \mathrm{d}\vec{l} = -\int_\infty^P \vec{E}\cdot \mathrm{d}\vec{l}$$

毕奥-萨法尔定律

- 线

$$\vec{B} = \frac{\mu_0}{4\pi}\oint_l \frac{I\mathrm{d}\vec{l}\times\vec{r}}{r^3}$$

- 面

$$\vec{B} = \frac{\mu_0}{4\pi}\iint \frac{\vec{i}\times\vec{r}}{r^3}\mathrm{d}S$$

- 体

$$\vec{B} = \frac{\mu_0}{4\pi}\iiint \frac{\vec{j}\times\vec{r}}{r^3}\mathrm{d}V$$

2　磁感应生电

$$\oint \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint \vec{B}\cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\Psi = \sum_{i=1}^n \Phi_i$$

$$\varepsilon = -\frac{\mathrm{d}\Psi}{\mathrm{d}t}$$

互感：线圈 1 的电流为 I_1，由它产生的磁场通过线圈 2 的全磁通为 Ψ_2

$$\Psi_2 = MI_1$$

线圈 2 的互感电动势为

$$\varepsilon_2 = -\frac{\mathrm{d}\Psi_2}{\mathrm{d}t} = -M\frac{\mathrm{d}I_1}{\mathrm{d}t}$$

自感

$$\Psi = LI$$

自感电动势

$$\varepsilon = -\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

变压器

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

3 物质中的静电场与静磁场、边界条件

静电场	静磁场
$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$	$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$
$\vec{D} = \varepsilon\vec{E}$	$\vec{H} = \frac{\vec{B}}{\mu}$
$\rho' = -\nabla\cdot\vec{P}$	$\vec{j}' = \nabla\times\vec{M}$
$\sigma' = \vec{P}\cdot\hat{n}$	$\vec{i}' = \vec{M}\times\hat{n}$
$\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0$	$\hat{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$
$\hat{n}\cdot(\vec{D}_2 - \vec{D}_1) = \sigma_f$	$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{i}_f$

4 静电与静磁能量和力的作用

自能互能

$U_i(\boldsymbol{r})$ 表示除第 i 个带电体外其余带电体在 $\boldsymbol{r}$ 处产生的电势

$U^{(i)}(\boldsymbol{r})$ 表示第 i 个带电体在 $\boldsymbol{r}$ 处产生的电势

$$W_{\text{自}} = \sum_{i=1}^{N}\iiint_{V_i}\rho(\boldsymbol{r})U^{(i)}\boldsymbol{r}\mathrm{d}V$$

$$W_{\text{互}} = \frac{1}{2}\sum_{i=1}^{N}\iiint_{V_i}\rho(\boldsymbol{r})U_i(\boldsymbol{r})\mathrm{d}V$$

电场

$$w_e = \frac{1}{2}\vec{E}\cdot\vec{D}$$

$$W_e = \frac{1}{2}\iiint\vec{E}\cdot\vec{D}\mathrm{d}V$$

磁场

$$w_m = \frac{1}{2}\vec{H}\cdot\vec{B}$$

$$W_m = \frac{1}{2}\iiint\vec{H}\cdot\vec{B}\mathrm{d}V$$

电容器

$$W = \frac{1}{C}\int_0^Q q\mathrm{d}q = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

建立电流过程中电源做得功

$$\varepsilon = iR + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$W = \int_0^T i^2 R \mathrm{d}t + \frac{1}{2}LI^2$$

5 稳恒电路、似稳电路（暂态和交流电）

$$\text{电路}\begin{cases}\text{稳恒电路}\\ \text{似稳电路}(l \ll \lambda)\begin{cases}\text{暂态电路}\\ \text{交流电路}\end{cases}\end{cases}$$

5.1 稳恒电路

$$\vec{j} = nq\vec{u}$$

欧姆定律的微观形式

$$\vec{j} = \sigma\vec{E}$$

焦耳定律的微观形式

$$p = \frac{j^2}{\sigma} = \vec{j}\cdot\vec{E}$$

电流连续性方程

$$\iint \vec{j}\cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}q}{\mathrm{d}t}$$

5.2 暂态电路

似稳电路 $l \ll \lambda$

似稳电路方程出发点为电荷守恒定律、电磁感应定律和欧姆定律

单一回路似稳方程

$$\varepsilon = iR + \frac{1}{C}\int i\mathrm{d}t + L\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i'}{\mathrm{d}t}$$

- R-L 电路
  - 充电

$$iR + L\frac{\mathrm{d}i}{\mathrm{d}t} = \varepsilon$$

  - 放电

$$iR + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

- R-C 电路
  - 充电

$$iR + \frac{q}{C} = \varepsilon \;\Rightarrow\; R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{C}q = \varepsilon$$

  - 放电

$$iR + \frac{q}{C} = 0 \;\Rightarrow\; R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{C}q = 0$$

- R-L-C 电路
  - 充电

$$iR + \frac{q}{C} + L\frac{\mathrm{d}i}{\mathrm{d}t} = \varepsilon$$
$$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{LC} = \frac{\varepsilon_0}{L}$$

	时间常数 τ
LR 电路	$\frac{L}{R}$
RC 电路	RC

5.3 交流电

- 函数描述

$$u(t) = U_0\cos(\omega t + \phi_u)$$
$$i(t) = I_0\cos(\omega t + \phi_i)$$
$$e(t) = \varepsilon_0\cos(\omega t + \phi_e)$$

- 矢量描述
- 复数描述

$$\tilde{V} = V_m e^{j(\omega t + \phi_u)}$$
$$\tilde{I} = I_m e^{j(\omega t + \phi_i)}$$
$$\tilde{\varepsilon} = \varepsilon_m e^{j(\omega t + \phi_e)}$$

$$u(t) = Re(\tilde{V})$$
$$i(t) = Re(\tilde{I})$$
$$e(t) = Re(\tilde{\varepsilon})$$
$$\varepsilon = iR + \frac{1}{C}\int i\mathrm{d}t + L\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i'}{\mathrm{d}t}$$

复阻抗

$$\dot{Z} = \dot{Z}_R + \dot{Z}_C + \dot{Z}_L + \dot{Z}_M$$
$$\dot{Z} = R + \frac{1}{j\omega C} + j\omega L + j\omega M$$

元件	电阻	电容	自感	互感
复阻抗 $\dot{Z}$	R	$\frac{1}{j\omega C}$	$j\omega L$	$j\omega M$
阻抗 Z	R	$\frac{1}{\omega C}$	ωL	ωM
辐角 ϕ	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$

串联

$$\dot{Z} = \dot{Z}_1 + \dot{Z}_2$$

并联

$$\frac{1}{\dot{Z}} = \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2}$$

功率因子

5.3.1 交流电路

- 串联谐振电路
  品质因素
  $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$
- 并联谐振电路
- 变压器电路 (理想变压器)
  - 无漏磁，即 $M^2 = L_1L_2$ 或 $\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$
  - 线圈无损耗，即 $R_1 = R_2 = 0$

6 电磁场动力学和电磁波

平面电磁波

$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\mu\varepsilon}\nabla^2\boldsymbol{E}$$

$$\frac{\partial^2 \boldsymbol{H}}{\partial t^2} = \frac{1}{\mu\varepsilon}\nabla^2\boldsymbol{H}$$

第二部分　模型

7　求电场的方法

7.1　库仑定律 + 叠加原理 (适用于只有少量电荷或矢量积分法容易求的带电体)

- 点电荷

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

- 体电荷

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho}{r^2}\mathrm{d}V\cdot\hat{r}$$

- 面电荷

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint\frac{\sigma}{r^2}\mathrm{d}S\cdot\hat{r}$$

- 线电荷

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint\frac{\lambda}{r^2}\mathrm{d}l\cdot\hat{r}$$

Examples:

- 电偶极子

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\vec{p} = q\vec{l}$$

  在远处，有

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} + \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p}\cdot\vec{r})}{r^5}\cdot\vec{r}$$

- 圆环

$$\mathrm{d}q = \lambda\mathrm{d}l = \lambda R\mathrm{d}\phi$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r^2}\cos\theta$$

$$\vec{E} = \frac{\lambda R\hat{z}}{2\varepsilon_0(R^2+z^2)^{\frac{3}{2}}}$$

- 圆盘

$$\mathrm{d}q = \sigma\mathrm{d}S = \sigma\mathrm{d}\mathrm{r}\cdot\mathrm{r}\mathrm{d}\phi = \sigma r\mathrm{d}r\mathrm{d}\phi$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r^2}\cos\theta = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]$$

$$\text{当 } z \gg R \text{ 时，} E = \frac{\sigma}{2\varepsilon_0}$$

- 半球面

$$\mathrm{d}q = \sigma R^2\sin\theta\mathrm{d}\phi\mathrm{d}\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r^2}\cos\theta = \frac{\sigma}{4\varepsilon_0}$$

7.2 高斯定理 (适用于电场分布高度对称情形)

$$\oiint \boldsymbol{D}\cdot \mathrm{d}\boldsymbol{S} = Q_f$$

- 球对称（如点电荷、均匀带电球面或球体）
- 轴对称（如无限长直导线、圆柱面、圆柱体）
- 面对称（如无限大平面、平板、均匀带电平板）

7.3 用电势求场强

$$\vec{E} = -\nabla U$$

7.4 几种特殊的电场

- $$\text{板} \propto \frac{\sigma}{\varepsilon_0}$$
- $$\text{线} \propto \frac{1}{r}$$
- $$\text{点电荷} \propto \frac{1}{r^2}$$
- $$\text{电偶极子} \propto \frac{1}{r^3}$$

8 求力

$$\vec{F} = (\nabla W_e)_U = -(\nabla W_e)_Q$$
$$\vec{F} = (\nabla W_m)_I = -(\nabla W_m)_\Phi = (\nabla W_m)_m$$

9 大 boss 偶极子

9.1 电偶极子

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$
$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} + \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p}\cdot\vec{r})}{r^5}\cdot\vec{r}$$

$$\vec{L} = \vec{p}\times\vec{E}$$

$$W_e = -\vec{p}\cdot\vec{E}$$

9.2 磁偶极子

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3} + \frac{\mu_0}{4\pi}\frac{3(\vec{m}\cdot\vec{r})}{r^5}\cdot\vec{r}$$
$$\vec{L} = \vec{m}\times\vec{B}$$

$$W_m = \vec{m}\cdot\vec{B}$$
$$\vec{F} = (\vec{m}\cdot\nabla)\vec{B} = \left[\nabla(\vec{m}\cdot\vec{B})\right]_m$$

$$W_m = -\frac{\mu_0}{4\pi}\frac{\vec{m}_1 \cdot \vec{m}_2}{r^3} + \frac{\mu_0}{4\pi}\frac{3}{r^5}(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot \vec{r})$$

两个磁偶极子间的相互作用力

$$\vec{F} = \frac{\mu_0}{4\pi}\frac{3\hat{e}_r}{r^4}(\vec{m}_1 \cdot \vec{m}_2 - 5m_{1r}m_{2r}) + \frac{\mu_0}{4\pi}\frac{3}{r^4}(m_{2r}\vec{m}_1 + m_{1r}\vec{m}_2)$$

10　求磁场的方法

10.1　毕奥萨法尔定律

- 线

$$\vec{B} = \frac{\mu_0}{4\pi}\oint_l \frac{I\mathrm{d}\vec{l} \times \vec{r}}{r^3}$$

- 面

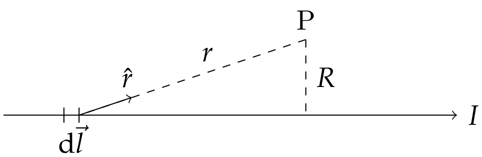
$$\vec{B} = \frac{\mu_0}{4\pi}\iint \frac{\vec{i} \times \vec{r}}{r^3}\mathrm{d}S$$

- 体

$$\vec{B} = \frac{\mu_0}{4\pi}\iiint \frac{\vec{j} \times \vec{r}}{r^3}\mathrm{d}V$$

Examples:

- 直导线



$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\mathrm{d}\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 - \cos\theta_2)$$

无限长直导线，$\cos\theta_1 = 1 \quad \cos\theta_2 = -1$

$$B = \frac{u_0 I}{2\pi R}$$

- 磁矩

$$\vec{m} = I\vec{S}$$

面积矢量

$$\vec{S} = \frac{1}{2}\oint \vec{R} \times \mathrm{d}\vec{R}$$

当 P 位于 z 轴上时，

$$B_x = 0 \qquad B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{\frac{3}{2}}}$$

当 P 远离线圈时

$$B_x = \frac{\mu_0}{4\pi}\frac{3m}{r^3}\sin\theta\cos\theta \qquad B_z = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3} + \frac{\mu_0}{4\pi}\frac{3m}{r^3}\cos^2\theta$$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3} + \frac{\mu_0}{4\pi}\frac{3\,(\vec{m} \cdot \vec{r})}{r^5} \cdot \vec{r}$$

类比电偶极子

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} + \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p} \cdot \vec{r})}{r^5} \cdot \vec{r}$$

- 螺线管

$$\mathrm{d}B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}} n\mathrm{d}z$$

$$B_z = \int_{z_1}^{z_2} \mathrm{d}B_z = \frac{\mu_0 n I}{2}(\cos\beta_2 - \cos\beta_1)$$

无限长螺线管

$$B_z = \mu_0 n I$$

半无限长螺线管的一段

$$B_z = \frac{1}{2}\mu_0 n I$$

10.2 安培定理

$$\nabla \times \vec{B} = \mu_0 \vec{J} \tag{1}$$

利用 Stokes 定理将其转化为积分形式

$$\int \left(\nabla \times \vec{B}\right) \cdot \mathrm{d}\vec{a} = \oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \int \vec{J} \cdot \mathrm{d}\vec{a} = \mu_0 \sum I \tag{2}$$

即有

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \sum I \tag{3}$$

- 无限长螺线管

$$B \cdot L = \mu_0 N I \;\Rightarrow\; B = \frac{\mu_0 N I}{L} = \mu_0 n I$$

- 螺绕环

$$B \cdot 2\pi r = \mu_0 N I \;\Rightarrow\; B = \frac{\mu_0 N I}{2\pi r}$$

11 求磁能

一个载流线圈的磁能

$$W_m = \frac{1}{2} I \Phi_m = \frac{1}{2} L I^2$$

N 个载流线圈系统的磁能

$$W_m = \frac{1}{2} \sum_{i,k=1\ i\neq k}^{N} M_{ik} I_i I_k + \frac{1}{2}\sum_{i=1}^{N} L_i I_i^2$$

载流线圈在外场中的磁能

$$W_m = \vec{m} \cdot \vec{B}$$

12 求自感系数的方法

- $$\varepsilon = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

- $$\Psi = LI$$

- $$W = \frac{1}{2}LI^2$$